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Load Diffusion in Composite Structures

FINAL REPORT (03/25/98--03/24/00)

C. O. Horgan and J. G. Simmonds

National Aeronautics and Space Administration

School of Engineering and Applied Science
University of Virginia
Charlottesville, VA 22903

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FINAL REPORT

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED
C. O. Horgan, J. G. Simmonds (PIs); T. McDevitt (Ph.D.), Zili Qian (Ph.D.), Alice M. Chan (M.S.).

9. REPORT OF INVENTIONS (BY TITLE ONLY): None

10 STATEMENT OF THE PROBLEMS STUDIED; MAIN RESULTS

Preliminary Remarks

This research has been concerned with *load diffusion* in composite structures. Fundamental solid mechanics studies were carried out to provide a basis for assessing the complicated modeling necessary for large scale structures used by NASA. An understanding of the fundamental mechanisms of load diffusion in composite subcomponents is essential in developing primary composite structures. Analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and assessing results from finite element analyses. The decay

behavior of stresses and other field quantities provides a significant aid towards this process. The results are also amenable to parameter study with a large parameter space and should be useful in structural tailoring studies.

Summary of Results :

This research was concerned with the general issues of *local gradients*, *discontinuities* and *load diffusion* in composite structures. Fundamental solid mechanics studies were carried out to provide a rational basis for assessing the complicated modeling necessary for large scale structures used by NASA. An understanding of the fundamental mechanisms and nature of load diffusion in composite subcomponents is essential in the development of primary composite structures, for example, the diffusion of wingbox loads into the composite fuselage shell wall of the proposed high-speed civil transport aircraft or the diffusion of internal loads around major discontinuities in stiffened fuselage shells such as passenger doors. Special purpose analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and assessing results from general purpose finite element analyses. For example, a rational basis is needed in choosing where to use three-dimensional to two-dimensional transition finite elements in analyzing stiffened plates and shells. The decay behavior of stresses and other field quantities furnished by the research carried out here provides a significant aid towards this element transition issue.

The particular problem area investigated was that of local gradients, discontinuities and load diffusion in anisotropic and composite structures. In the application of elasticity theory to problems of practical interest, an essential simplification is made by ignoring local gradients or discontinuities through consideration of load resultants. For example, the theories for strength of materials, involving beams, plates and shells have such relaxed boundary conditions as cornerstones of their development. The justification for such approximations is usually based on some form of Saint-Venant's principle characterizing the boundary layer behavior involved. Thus, Saint-Venant's principle is appealed to in neglecting *local gradients* and *discontinuities*, and experience with homogeneous *isotropic* materials (e.g., metals) in linear elasticity has served to establish this standard procedure. It lies at the very foundations of applied structural analysis as practiced in the aerospace industry. Saint-Venant's principle also is the fundamental basis for static mechanical tests of material properties. Thus property measurements are made in a suitable *gage section* where *uniform* stress and strain states are induced and local effects due to clamping of the specimen are neglected on invoking Saint-Venant's principle. Such traditional applications of Saint-Venant's principle require major modifications when strongly anisotropic and composite materials are of concern. For such materials, local stress effects persist over distances *far greater* than is typical for isotropic metals. The implications of such extended end zones due to anisotropy are far reaching in the proper analysis and design of structures using advanced composite materials.

In [1, 2, 5] the purpose is to further investigate the effects of material inhomogeneity and the combined effects of material inhomogeneity and anisotropy on the decay of Saint-Venant end effects. Saint-Venant decay rates for self-equilibrated edge loads in symmetric sandwich structures are examined in the context of anti-plane shear for linear anisotropic elasticity. The most general anisotropy consistent with a state of anti-plane shear is considered, as well as a variety of boundary conditions. Anti-plane or longitudinal shear deformations are one of the simplest classes of deformations in solid mechanics and characterize Mode III fracture. The resulting deformations are completely characterized by a single out-of-plane displacement which depends only on the in-plane coordinates. They can be thought of as complementary deformations to those of plane elasticity. It is shown in [1, 2, 5] that material inhomogeneity significantly affects the practical application of Saint-Venant's principle to

sandwich structures. Applications to specific sandwich configurations used by the Boeing Company are presented in [1, 2, 5]. The effects of imperfect interface bonding are examined in [2]. Several configurations of importance in aircraft design are considered in [2]. In [3] results are obtained on the decay of end effects in continuously inhomogeneous elastic materials. Such functionally graded materials (FGMs) are now used in a variety of technological applications e.g. to provide superior oxidation and thermal shock resistances. Thermal residual stresses can be relaxed in metal-ceramic layered material by inserting a functionally graded interface layer between the metal and the ceramic. It is shown in [3] that the inhomogeneity can enhance or inhibit load diffusion. The results provide analytical guidelines for material tailoring and have clear NASA relevance. Other problems for FGMs have been investigated in [6-9]. In [6], the torsion problem for functionally graded elastic bars has been considered. It is shown that, in contrast to the classical isotropic case, the maximum shear stress can occur in the interior of the cylinder. Implications for material failure are immediate. Vibrations of inhomogeneous strings, rods and membranes are considered in [7]. Some closed-form exact solutions are developed in [7], which are useful as benchmark problems to assess the accuracy of numerical schemes. It is also shown that an integral-equation based technique is very accurate in obtaining lower bounds for the fundamental frequency. The analog of the classic Lamé problem for an internally or externally pressurized hollow cylinder is examined in [8] for the case on an inhomogeneous isotropic elastic solid. It is shown that the stress response of the inhomogeneous tube is significantly different from that of the homogeneous body. For example, the maximum hoop stress does not, in general, occur on the inner surface in contrast with the situation for the homogeneous material. The results are illustrated using a specific radially inhomogeneous material model for which explicit exact solutions are obtained. This fundamental study should have wide applicability to complicated pressure vessel configurations employed by NASA. In [9], the stress response of a solid inhomogeneous disk, rotating with constant angular velocity about its center, is examined. Again, the stress response is radically different from that of the homogeneous disk. For example, the maximum radial and hoop stresses no longer occur at the center, as they do for the homogeneous material. It is also shown how the material inhomogeneity may be tailored so that the radial and hoop stress are identical throughout the disk.

In [4], a comprehensive invited review book chapter on the research on Saint-Venant end effects has been completed and will be published early in the year 2000 in a 6 volume book series. This set, entitled "Comprehensive Composite Materials", will review the developments in composites technology in the century just ending and set the stage for the new millennium.

The second edition of the book [15] by Libai and Simmonds, *The Nonlinear Linear Theory of Elastic Shells* contains a wealth of up-to-date material and references on various aspects of the nonlinear theory of shells and membranes. This should provide a standard reference for workers around the world in this field.

Although he retired from the University of Virginia in May, 1998, Professor Simmonds remains active in research. In the spring and summer of 1999 he spent seven months in Paris, working (again) with Pierre Ladevèze, who is director of the Laboratoire de Mécanique et Technologie. Much technical information was interchanged about problems of concern to both the U.S. and the French aerospace industry.

The work [10-23] of Professor Simmonds and his collaborators has concerned all aspects of thin-walled structures. Recently, at the request of Dr. Nemeth of LARC, Professor Simmonds has undertaken an extensive study of the beam-like behavior as well as decay effects in fully anisotropic general cylindrical shells. One paper has appeared, one is to be published, and two are nearly ready for publication. It is hoped that these results, many of which are in the form of simple analytical formulas, will have great technological impact for they allow the designer to minimize the unavoidable slowly

decaying internal stresses coming from edge disturbances by suitably choosing material parameters.

NASA Contact

- (1) Continued close contact maintained with Dr. J. Starnes and Dr. M. Nemeth, NASA.
- (2) Letters and reprints sent to several NASA personnel.
- (3) J. G. S. presented an invited lecture entitled "Some comments on the status of shell theory at the end of the 20th century: complaints and correctives," at the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Kissimmee, Florida, April 7-10, 1997.
- (4) Extensive contact with Prof. Daniel Inman, Director, Center for Intelligent Material Systems and Structures, Virginia Tech on potential applications of the work to smart structures technology. Prof. Horgan presented a seminar at Virginia Tech on March 19, 1997 which has led to sustained interaction between their research groups. The importance of end effects in smart structural damping problems e.g. layered beams and plates with viscoelastic, metallic and PZT components is currently under investigation and promises to have a major impact on NASA technology.
- (5) Professors Horgan and Simmonds attended a briefing at the NASA Structural Mechanics Division, NASA Langley AFB, Virginia on May 28, 1998 concerning relevance of the work to the Aging Aircraft Structures Program. They also attended the Second Joint NASA/FAA/DoD Conference on Aging Aircraft, Williamsburg, Va, August 31-Sept 2, 1998.
- (6) C. O. H. presented an invited lecture entitled " Saint-Venant end effects for sandwich structures" at the 4th International Conference on Sandwich Construction, Royal Institute of Technology, Stockholm, Sweden, June 1998.
- (7) Professor Simmonds presented an invited keynote lecture entitled : " The status of shell theory at the end of the millennium " at the 6th Conference on Shell Structures, Theory and Applications, Gdansk, Poland, October 1998.
- (8) Professor Horgan presented the paper entitled " Structural Integrity of sandwich panels : edge effects" at the 15th U.S. Army Symposium on Solid Mechanics, Myrtle Beach, S. C., April 12-14, 1999.
- (9) Professor Horgan presented the lecture " Anisotropy induced singularities in linear elasticity" as a keynote lecture at the symposium in honor of J. L. Nowinski, University of Delaware, May 1999.
- (10) Professor Horgan presented the invited lecture " Anisotropy induced singularities in linear elasticity" at a Special Session on "Recent Developments in Anisotropic Elasticity" at the 1999 ASME Mechanics and Materials Conference, Virginia Tech., Blacksburg, VA, June 1999 and at the Fourth International Congress on Industrial and Applied Mathematics, (ICIAM IV), Edinburgh, Scotland, July 1999.
- (11) Professor Simmonds gave a lecture entitled, "A simple, logical, comprehensive approach to non-linear shell theory," at the 35th Annual Meeting of the Society of Engineering Science, Washington State University, Pullman, Sept. 1998.
- (12) Professor Simmonds presented a lecture at the Fourth International Congress on Industrial and Applied Mathematics in Edinburgh in July 1999 entitled, "Computing exact elastodynamic linear three-dimensional solutions for plates from classical two-dimensional solutions."

Technology Transfer

The results of this research are being widely utilized in the technical literature on composite materials, with technology transfer to such areas as composite design and materials testing. The attached summary of technology transfer to the Boeing Commercial Airplane Group attests to the applicability of the results. Dr. Horgan visited the Boeing Company in Seattle on May 25, 1995 to further this important area of technology transfer. He presented a 1-hour lecture entitled *End Effects in Composite Structures* to the Boeing group. This interaction continues (see letter attached, July, 1996). Further contact with Boeing personnel was made at the 4th International Conference on Sandwich Construction, Stockholm, Sweden in June 1998. Interaction with Dr. Tom Bitzer, Hexcel Corporation, also took place at this meeting. Dr. Bitzer is chair of the ASTM Committee on Testing of Composites. Dr. Horgan has also initiated joint work with Professor Leif Carlsson of Florida Atlantic University on the role of end effects in composite testing. Professor Carlsson visited the University of Virginia in October 1998 and presented a seminar on issues in composite testing. Professors Horgan and Carlsson are collaborating on a book chapter to appear in the 6 volume *Comprehensive Composite Materials* (ed. by A. Kelly and C. Zweben), Elsevier Publishers, 2000 (in press). This book series will review the developments in composites technology in the century just ending and set the stage for that to come.

Professors Horgan and Simmonds have presented numerous invited colloquia on their NASA supported work at several universities in the U. S. and Europe and at national and international meetings. Between them, they serve on the editorial boards of six major journals, namely, *Applied Mechanics Reviews*, *International Journal of Non-Linear Mechanics*, *Journal of Elasticity*, *SIAM Journal on Applied Mathematics*, *Mathematics and Mechanics of Solids*, and the *Quarterly of Applied Mathematics*. They are both fellows of the American Society of Mechanical Engineers and the American Academy of Mechanics. Dr. Horgan served on the Board of Directors, Society of Engineering Science from 1994-1999.

Statement of the Impact of the Proposed Research

The research program carried out here contributes to NASA's Structural Mechanics Program in several ways. At the outset it has two important, seemingly paradoxical, impacts. Firstly, it shows from specific problems, simple enough to be amenable to considerable analysis yet elaborate enough to be of practical significance, that end and edge effects in anisotropic, laminated, materials and structures are far more severe than in homogeneous isotropic structures of the same geometry and under the same loads. And secondly, it shows that *elementary* (classical) theories of beams, plates, and shells, *properly supplemented and iterated*, suffice for the accurate determination of stresses in thin-walled, layered structures, even in the presence of strong end or edge effects. Thus, an important impact of this research will be to convince users of large structural computer codes that the underlying equations must be examined carefully to make sure, on the one hand, that they properly and accurately incorporate end and edge effects, and, on the other, that they are not unnecessarily elaborate (i.e., incorporate effects of the same order as others they tacitly neglect.)

The results of this research also assist in the development of new structural concepts, using composite materials, for application to primary aircraft structures. The analysis is amenable to parametric studies of value in structural tailoring. Analytic results of the type obtained here are crucial complements to large scale computational analyses of actual aircraft and space structures that often have several local gradients and discontinuities involving load diffusion.

A Technology Transfer Example

A Boeing/NASA Advanced Technology Composite Aircraft Structures (ATCAS) Program has been active since 1989. The primary objective of this program is to:

"Develop an integrated technology (manufacturing & structures) and demonstrate a confidence level that permits cost-and weight-effective use of advanced composite materials in primary structures of aircraft with the emphasis on pressurized fuselages."

In this program, a section of a widebody aircraft (244" dia) just aft of the wing/body intersection is being analyzed by the Boeing Commercial Airplane Group in Seattle, Washington. Sandwich structures are being used for the side and keel of this section. The particular structures consist of Hercules' AS4/8552 for the skin and Hexcel's HRP honeycomb core (see next page for details on layup etc.). Compression testing of laminate coupons indicate the need to incorporate Saint-Venant end effects in interpretation of the test data. The work of the PI's is being utilized in this effort. One of the P.I's (C. O. H.) visited the Boeing Group in Seattle on July 1, 1994 and on June 1/2, 1995 to consolidate this interaction. Collaborative research with the Boeing scientists (Dr. W. A. Avery, coordinator) is being initiated.. One objective is to develop a systematic testing program to be carried out by Integrated Technologies, Inc. (Intec), Bothell, WA, under subcontract to Boeing. Preliminary tests by Intec have indicated problems due to end effects in the sandwich panels under investigation. It is anticipated that the results obtained in our research program will have direct application to these problems. In fact, the interaction with the Boeing/Intec mechanics and materials group is providing additional motivation and stimulus to our efforts in understanding the extent of Saint-Venant end effects in advanced composite materials and structures.

Table 1 Material Types

Panel ID	Skin Material	Form	No of Plies	Nominal Ply Thickness (in)	Layup ID	Core Type	Core Thickness (in)
AK7	8-256	Tow	12	0.0080	Keel 1	HRP-3/16-8.0	0.75
AK8	8-256	Tow	12	0.0080	Keel 1	HRP-3/16-8.0 & TPC-3/16/5.5	0.75
AK10a	AS4/8552	Tape	12	0.0073	Keel 1	HRP-3/16-8.0	0.75
AK10b	AS4/8552	Tape	12	0.0073	Keel 3	HRP-3/16-8.0	0.75
AK10c	AS4/8552	Tow	12	0.0073	Keel 1	HRP-3/16-8.0	0.75
AK10d	AS4/8552	Tow	12	0.0073	Keel 1/ Keel 3	HRP-3/16-8.0	0.75

Table 2 Layups

Layup ID	Number of Plies	Ply Orientation
Keel 1	12	[45/0/-45/90/0/-45/45/0/90/-45/0/45]
Keel 3	12	[30/-30/0/90/0/-45/45/0/90/0/-30/30]

July 5, 1996
BYH20-BFB-L96-043

Dr. Arje Nachman
Program Director, Applied Analysis
AFOSR, 110 Duncan Ave., Suite B115
Bolling AFB
Washington, DC 20332-0001

BOEING

Dear Dr. Nachman,

A few days ago Cornelius Horgan from the University of Virginia asked me to write you and explain the relevance of his work to Boeing. I am happy to do so. This letter is the response to that request.

In 1989 Boeing started work on the Advanced Composite Technology Aircraft Structure (ATCAS) program, which is funded by NASA's Advanced Composite Technology (ACT) initiative. In this program Boeing has been developing the materials, structures, and manufacturing technology for a composite fuselage for a widebody aircraft. The goal is to design the structure such that there is significant savings in both cost and weight.

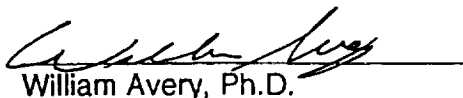
Early in the program we identified sandwich structure as having a high potential to save cost because the tooling and manufacturing processes for skin-stringer structures are expensive. Consequently we baselined sandwich construction for the fuselage keel and side panels.

When we started collecting our structural database we compression tested several solid laminate and sandwich coupons, many of them with holes. The failure loads were higher than expected, and we realized that we may have not been getting the full stress concentrations at the edges of the holes. This was confirmed through some photoelastic analyses of additional coupons. Part of the problem was test method related. But a review of some of Dr. Horgan's work helped us understand how St. Venant's effects are different for anisotropic and sandwich structures. His work became useful in guiding us in sizing test coupons based on degree of anisotropy and the particular configuration of the sandwich structure. Essentially, we found that we needed a longer specimen in order to get uniform load into the test coupon. Dr. Horgan's analyses helped us quantify that increase in length.

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I appreciate Dr. Horgan's effort to keep in touch with Boeing and solicit ideas for research. I find him very "customer oriented". His work has been helpful to us. If you have any questions please feel free to contact me.

Regards,



William Avery, Ph.D.
Principal Engineer
Composite Structures Development
M/S 6H-CR
(206) 234-0444
e-mail: william.b.avery@boeing.com

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Appendix : A Plane Stress to Plane Strain Transition Issue in Fracture Mechanics

At a meeting of the P.I.s with Drs. Starnes and Nemeth at LARC on May 28, 1998, an important issue of relevance to NASA and the Aging Aircraft Structures Program was discussed. The particular problem addressed is depicted in the attached figure, supplied by Dr. Starnes. In the shell structure shown, subject to combined load, the usual procedure adopted in finite-element analyses is to use a three-dimensional analysis (or generalized plane stress analysis) near the crack tip and a plane strain analysis "far enough away" from the crack-tip. The open issue is to quantify the length over which this transition should take place. The research program carried out by the P.I.s over several years is relevant to this general question of transition from a local three-dimensional framework to a global two-dimensional one.

Before tackling the problem for a shell, we have investigated the literature on the same issue for a crack in a flat plate and found three significant publications. An approximate analytical approach [1] developed by Yang and Freund in 1985 was followed by an experimental study [2] by Rosakis and Ravi-Chandar in 1986. The main conclusions drawn in [1, 2] are that three-dimensional effects are important within a zone ahead of the crack tip of the *order of one-half of the plate thickness*. A rigorous analytical investigation of this problem for a plate, based on the transition from three-dimensional elasticity to two-dimensional theory was carried out very recently (1998) by Knowles [3]. In [3], it is shown how the issue may be formulated in terms of a Saint-Venant type principle. The energy decay methods that have been developed to study end effects and that have been widely used by the P.I.s are then applied in [3] to confirm and substantiate the results of [1, 2]. The mathematical analysis required in this work [3] is formidable and it is pointed out in [3] that some improvements in the results are desirable . Nevertheless, our preliminary research indicates that the approach of [3] may be feasible to tackle the shell problem and it is hoped to carry out this work under continued NASA support.

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MEASURED AND PREDICTED TEARING RESPONSE

